Exam "Toepassingen van de Quantum-Fysica" Groningen, 5-3-99

Please each problem on a separate sheet of paper (to facilitate correction by different people). Please put your name on each sheet!

Problem 1 *Hyperfinestructure of hydrogen.*

The spins of electron and proton are vectorial quantities, characterized by quantum numbers $s=1/2$ and $I=1/2$.

a) which are the resulting values for the total angular momentum F of the groundstate hydrogen atom?
b) give arguments, why states with different F have different energies.
c) indicate in a drawing how the energies split and vary when an external magnetic field **B** is applied; indicate in this drawing also the quantum numbers which are relevant in case of strong magnetic fields.
d) discuss, how one could experimentally obtain an ensemble of atoms in which the H-atoms are in the state with higher energy.

Problem 2 *Laser Stabilisation via Zeeman effect*

We want to stabilize the frequency output of a laser such that it is resonant with the Na(*3s-3p*) transition at $\lambda = 600$ nm. To that end we take part of the output to irradiate a Na vapour cell. Fluorescence of the vapour takes place as soon as the frequency is somewhere within the Doppler profile of the resonance line.

a) give an indication how broad the frequency range is, in which fluorescence is observed. Assume a temperature of 300 °K. (atomic mass of Na is 23, proton mass $m_p=1.67\ 10^{-27}$ kg, Boltzman constant $k=1.38\ 10^{-23}\ JK^{-1}$)

We want to stabilise the laser with an accuracy of several kHz.

b) why is that practically impossible by simply observing the fluorescence?

To improve the accuracy we place the vapour cell in a magnetic field and split the laser output into two individual beams, one of which is given right-hand and the other left-hand circular polarization. These two beams are - spatially separated - irradiated into the vapour cell. The frequency-dependent fluorescence of the vapour is measured with photodiodes, e.g. in a direction perpendicular to the laser light. Due to the Zeeman effect these signals exhibit a frequency shift with respect to each other.

c) sketch qualitatively the two signals as well as their difference as a function of frequency.
d) For a magnetic field of $B=10^{-2}$ Tesla calculate the frequency shift, assuming that the fluorescence is exclusively due to the D_2 line (the $^2S_{1/2}$-$^2P_{3/2}$ transition); disregard the influence of the nuclear spin.
($\mu_B=9.27*10^{-24}\ Am^2$, $g_j= 1+[j(j+1) + s(s+1) - l(l+1)]/[2\ j(j+1)]$, $h=6.6\ 10^{-34}$ Js)
e) estimate the magnetic field strength necessary to "catch" the laser frequency and to stabilize to the zero-crossing of the difference signal as soon as reasonable fluorescence occurs.
f) estimate the fieldstrength necessary to bring the absorbtion of a Na-flame out of absorption resonance for light from a Na discharge lamp.

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Problem 3 — *Allowed terms and Hund's rules*

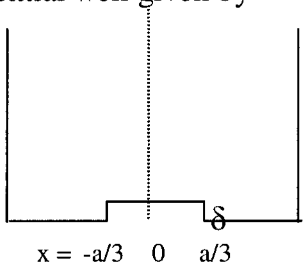
Consider an atomic state with two electrons in the configuration (2p3p) and ($2p^2$).
Discuss for these two cases

a) what are the possible values for the total orbital angular momentum L?
b) what are the possible values for the total spin S?
c) which terms and states are allowed
d) what is their energetic ordering according to Hund's rules?

Problem4 — *Perturbation*

Consider the one-dimensional square 'stepped' potential well given by

$V(x) = +\infty$ for $|x|>a$
$V(x) = 0$ for $a/3 < |x| < a$
$V(x) = \delta$ for $|x| < a/3$
where δ is small with respect to the energy of the lowest level



a) what are the lowest wavefunctions for the undisturbed potential (δ=0)
b) what are the corresponding eigenenergies?
c) discuss, why first order perturbation theory is appropriate to determine energies and wavefunctions for the disturbed potential
d) from first order perturbation theory find the two lowest energy levels,
e) give the general expression for the corresponding new wavefunctions. Are some terms vanishing in these expressions?